Doubly heavy hadrons and the domain of validity of doubly heavy diquark–anti-quark symmetry

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In the limit of heavy quark masses going to infinity, a symmetry is known to emerge in QCD relating properties of hadrons with two heavy quarks to analogous states with one heavy anti-quark. A key question is whether the charm mass is heavy enough so that this symmetry is manifest in at least an approximate manner. The issue is crucial in attempting to understand the recent reports by the SELEX Collaboration of doubly charmed baryons. We argue on very general grounds that the charm quark mass is substantially too light for the symmetry to emerge automatically via colour coulombic interactions. However, the symmetry could emerge approximately depending on the dynamical details of the non-perturbative physics. To treat the problem systematically, a new expansion that simultaneously incorporates NRQCD and HQET is needed.

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INTRODUCTION

It has been known for some time that in the limit of arbitrarily large heavy quark masses that QCD has a symmetry which relates hadrons with two heavy quarks (anti-quarks) to analogous states with one heavy anti-quark (quark) [1]. We will refer to this symmetry as the doubly heavy diquark–antiquark (DHDA) symmetry. Presumably when the masses are finite, but very large, a remnant of this DHDA symmetry will survive in the form of an approximate symmetry. A key issue is how large must the masses be before such an approximate DHDA symmetry is manifest in a useful way. The issue is particularly relevant for charm quarks—both because the charm quark is the lightest of the heavy quarks and hence the approximation is most likely to fail and because doubly bottomed hadrons (or hadrons with a charm and a bottom) are presumably more difficult to create and detect than doubly charmed ones.

The issue remained of only marginal importance in the absence of observed doubly heavy hadrons. However, in the past several years, the SELEX Collaboration has reported the first sighting of doubly charmed baryons [2]. Four states, $\Xi_{cc}^{+}(3443)$, $\Xi_{cc}^{++}(3460)$, $\Xi_{cc}^{+}(3520)$, and $\Xi_{cc}^{++}(3541)$ (which have been interpreted as two pairs of iso-doublets) are reported, as shown in Fig. 1. It should be noted that all four states were identified through their weak decay products. This is surprising as one would ordinarily expect the excited states to decay electromagnetically much more rapidly and thus wash out a signal for weak decays. This issue creates a potential problem for any interpretation of the data. Additionally, most recently, BaBar has reported that they have not observed any evidence of doubly charmed baryons in e^+e^- annihilations [3]. However, we would set these issues aside and take the existence of all four states as given to ask whether the properties of these states could be understood at least qualitatively in terms of the DHDA symmetry. Recently refs. [4] and [5] argued that the splitting between the lower doublet and the upper doublet Ξ states can be understood semi-quantitatively (at the 30% level) in terms of an approximate DHDA symmetry.

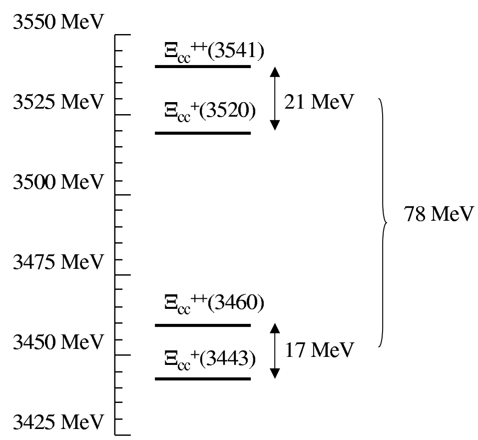

FIG. 1: Spectrum of Ξ_{cc} that have been observed by the SELEX Collaboration [2].

This paper critically examines the extent to which an approximate DHDA symmetry could be present for charm quarks. This is of importance both for the doubly charmed states found by SELEX and also for the existence of putative doubly charmed tetraquarks which are known to exist in the heavy quark limit [6] and in potential models [7]. We find strong evidence to suggest that the charm quark mass is *not* heavy enough for the symmetry to emerge automatically of color coulombic interactions. The key issue is the degree to which scales that separate in the heavy quark limit (and whose separations are critical to the derivation of the DHDA symmetry) in fact separate for doubly charmed systems. As we will detail below, such a scale separation probably does not hold. Despite this, we will show that the presence of certain non-perturbative interactions could result in an approximate DHDA symmetry in the charm sector.

To begin the discussion, let us consider why one expects the DHDA symmetry. Physically, it arises from a diquark pair forming a tightly bound nearly point-like object. The attraction between the two heavy quarks in the diquark comes from a color coulombic interaction that is attractive in the color $\bar{\mathbf{3}}$ channel. If the mass of the quarks is large enough, the heavy quarks move slowly and act like non-relativistic particles in a coulombic potential. As the size of a coulombic bound state is inversely proportional to its mass (for fixed coupling), in the large mass limit the diquark becomes a heavy, small object with color $\bar{\mathbf{3}}$. To a good approximation it becomes a static point-like $\bar{\mathbf{3}}$ color source; in this sense it acts in essentially the same way as a heavy anti-quark. This symmetry was first discussed by Savage and Wise [1] in the context of relating the properties of doubly heavy baryons, QQq, to those of heavy mesons, $\bar{Q}q$.

To the extent that one can treat the heavy diquark as formed, one can simply use standard heavy quark effective theory (HQET) to describe the properties of the doubly heavy baryons. Since the diquark in the doubly heavy baryon essentially acts as an antiquark, one can directly relate the properties of this system to heavy mesons. Using the HQET effective Lagrangain in ref. [1], a relationship valid at large M_Q for the mass difference of spin excited states between the doubly heavy baryons and heavy mesons was derived [19]:

$$m_{\Sigma^*} - m_{\Sigma} = \frac{3}{4}(m_{P^*} - m_P), \tag{1}$$

where Σ and Σ^* are the doubly heavy anti-baryons with $S = \frac{1}{2}$ and $S = \frac{3}{2}$, respectively, and P and P^* are the heavy mesons with $S = 0$ and $S = 1$, respectively. From the prospective of HQET, this relationship should hold to $O(\Lambda_H^2/m_Q)$ where Λ_H is a typical hadronic energy and is proportional to, but not identical to Λ_{QCD}. However as was discussed in ref. [4], the finite size of the diquark gives rise to corrections formally larger than this in the large mass regime. At the time of the Savage and Wise paper, this relationship was a prediction of the theory: doubly heavy baryons had not been discovered. The SELEX data will allow us to explore this relation with some real world data.

Before proceeding further, we should note that this analysis is based on the assumption that a spatially small and tightly bound diquark configuration exists and remains unexcited in the dynamics. The key question we address is the extent to which this assumption is true.

To examine the issue of diquark excitations, a systematic treatment for the dynamics of two heavy quarks is needed. At a formal level the non-relativistic expansion of the heavy quark degrees of freedom with QCD (NRQCD) is the natural language to explore this issue. NRQCD was first developed by Bodwin, Braaten, and Lepage [9], where it was modeled after a similar treatment in the context of QED [10]. HQET is generally considered as an expansion in powers of p/m, with $p \sim \Lambda_H$. Thereby it creates two energy scales, m and Λ_H. On the other hand, NRQCD requires the introduction of two new scales: the characteristic momentum, mv, and energy scale, mv^2, where $v \sim \alpha_s(mv)$ is the characteristic velocity of the two heavy quarks relative to each other. With the hierarchy, $m \gg mv \gg mv^2$, the characteristic regimes in terms of (energy, momentum) of the heavy quarks are: (m, m), (mv, mv), (mv^2, mv), and (mv^2, mv^2). These are conventionally referred to as hard, soft, potential, and ultrasoft, respectively. Traditional NRQCD has been further simplified into two different effective theories, pNRQCD and vNRQCD. pNRQCD integrates out the soft momentum gluons to form heavy diquarks states with definite color, and uses these diquark states as the degrees of freedom [11]. On the other hand, vNRQCD keeps the heavy quarks as explicit degrees of freedom while matching the effective theory at the hard scale [12]. In all forms of NRQCD, the separation of scales creates an expansion of powers of v.

On physical grounds, one expects that the NRQCD at leading order of systems with two heavy quarks (or anti-quarks) ought to reduce to the HQET description of the dual problem—*i.e.*, the problem related by the DHDA symmetry. Recently, ref. [4] derived the presence of DHDA symmetry in the context of pNRQCD while ref. [5] confirmed this for vNRQCD by showing the equivalence between vNRQCD and pNRQCD. It should be noted that this derivation represents a qualitatively new domain for NRQCD. Traditionally, NRQCD is applied to systems with one heavy quark and one heavy anti-quark with no valance light quark degrees of freedom. The fact that the technique may be extended to problems with two heavy quarks plus additional light quark degrees of freedom is non-trivial. One central point, that should be stressed, is that the derivation is quite general and applies equally well to the problem of heavy tetraquarks as well as doubly heavy baryons. The key advantage to the NRQCD formalism is that corrections to this symmetry can be systematically incorporated by working at higher order.

While it is known that the DHDA symmetry must emerge in the heavy quark limit, it is not immediately clear how large the corrections to the symmetry results should be for the realistic case in which heavy quarks have large but finite mass. Clearly the fundamental issue is the interplay between the diquark binding into an approximately point-like object and the extent that the diquark is point-like from the light quarks perspective; thus both the details of the physics of the interactions between the two heavy quarks as well as the between the heavy and light quarks are essential. Previous work in this area, [4, 5], have concentrated their efforts on perturbative expansions of the interactions between the two heavy quarks in the framework of NRQCD, and have not dealt with heavy/light interactions. Since the interactions between the heavy and light quarks are intrinsically non-perturbative, it cannot be estimated directly via the techniques of NRQCD. The full expansion should be a combination of HQET and NRQCD that incorporates the mixing of perturbative and non-perturbative scales. The issue of how to attack the question of the scale of these corrections for charmed or bottom quarks is the motivation for this paper. We do this in the context of the SELEX data with tools motivated by NRQCD. Even though we do not fully formulate the new combined expansion in this paper, we provide strong arguments suggesting the need for such a theory when dealing with doubly heavy mesons. This paper explores this issue both in terms of systematic treatment of the problem based on power counting in effective field theories and in terms of more heuristic phenomenological reasoning.

We divide this paper into two major sections. In the first, we work in the large quark mass limit, and develop the consequences of the spectrum in this regime. In the second section, we work with a finite quark mass and present arguments that show the SELEX data is not consistent with the large mass limit, the need for a new expansion to describe this system, and the justification beyond NRQCD of the apparent DHDA symmetry seen by SELEX.

CONSEQUENCES OF DHDA SYMMETRY IN THE LARGE MASS LIMIT

Before addressing the key question of whether the charm quarks are too light for the DHDA symmetry to be manifest, it is useful to consider just what implications the DHDA symmetry has on the spectrum when the symmetry is manifest—namely, when the quarks are sufficiently heavy. We attempt to consider the extreme limit, where all relevant scales cleanly separate. It is unlikely that the physical world exists in this limit. Nevertheless, an understanding of the the physics in this extreme regime is useful in understanding the applicable expansions. There has been extensive work using a variety of models in detailing the hadronic spectrum including [13] and [14] among others. Our focus here will be considering the spectrum in the context of a possible DHDA symmetry. We will consider a more modest regime, that is intended to describe the physical world, in the next section.

The first consequence we consider is qualitative—namely, the existence of exotic states. The DHDA symmetry in HQET was first used to relate doubly heavy baryons to heavy mesons [1]. However, the symmetry is independent of the light quarks in the problem. Formally, in NRQCD, the light quarks are govern by non-perturbative dynamics, and are thereby considered irrelevant when focusing on the heavy quarks in the large mass limit. As the DHDA symmetry applies in the heavy quark limit independent of the number and state of spectator light quarks, it is sufficient to consider an ordinary heavy baryon, Qqq. From DHDA symmetry, this state is directly related to a doubly heavy tetraquark state, $\bar{Q}\bar{Q}qq$. Thus in the heavy quark limit, when the DHDA symmetry is exact, the existence of heavy baryons implies the existence of doubly heavy tetraquarks.

The fact that doubly heavy tetraquarks must exist in the heavy quark limit has been shown previously. This was done both based on the simple argument discussed here and in the context of an illustrative model based on pion exchange [6, 7]. It should be noted that while being in the regime of validity of DHDA requires the existence of doubly heavy tetraquarks, the converse is not true: doubly heavy tetraquarks could be formed via other mechanisms. Nevertheless, the general result is significant in that the tetraquark has manifestly exotic quantum numbers in the sense that it cannot be made in a simple quark model from a quark–anti-quark pair. The observation of exotic hadrons has been a longstanding goal of hadronic physics. The prediction of the existence of an exotic particle directly from QCD, albeit in a limit of the theory, is of theoretical importance in that by direct construction QCD is compatible with exotics. Other exotic particles, such as a heavy pentaquark, have also been shown to exist in the heavy quark limit combined with the large N_c limit [15].

Let us now turn to more quantitative issues associated with the excitation spectrum. As noted in the introduction, the formal treatment of this problem incorporates NRQCD (for the interactions between the heavy quarks) and HQET (for the interactions between the heavy particles and the light degrees of freedom). The DHDA symmetry requires each of these effective theories to be in its domain of validity. In the heavy quark limit where both expansions will work, one has

$$M_Q \gg vM_Q \gg v^2 M_Q \gg \Lambda_H \gg \frac{\Lambda_H^2}{M_Q} \qquad (2)$$

where Λ_H is a typical hadronic scale proportional to Λ_{QCD} and v, the relative velocity of the heavy quark, is typically of order α_s and hence depends logarithmically on the quark mass. It should be noted that the NRQCD formalism is still valid for $M_Q v^2 \sim \Lambda_H$ as indicated by ref. [4]. However, none of the analysis in this work depends on $M_Q v^2$ being larger than Λ_H, and hence is consistent with the domain of validity on NRQCD. The formalism of NRQCD and its associated power counting rules remains valid for two heavy quarks in the color $\bar{\mathbf{3}}$ in the presence of additional light quark degrees of freedom and not just for heavy quark–anti-quark systems in the color singlet in heavy mass limit. This was shown in ref. [4] and verified in ref. [5].

It is important to note that these effective theories have different types of excitations with qualitatively different scales. Doubly heavy hadrons (in the formal limit of very large quark mass) have three characteristic types of excitation:

a. Excitations of order Λ_H^2/M_Q which correspond to the interaction of the spin of the diquark with the remaining degrees of freedom in the problem.

b. Excitations of order Λ_H which correspond to the excitations of the light degrees of freedom.

c. Excitations of order $v^2 M_Q$ which correspond to the internal excitation of the diquark.

The first two types of excitations can be understood in terms of HQET while the third requires NRQCD. The essential point is that as $M_Q \to \infty$ the three scales separate cleanly. Since these excitations all occur at disparate scales, they do not influence each other.

DHDA symmetry imposes many relations on the various types of excitations of various doubly heavy hadrons and their associated singly heavy ones. To enumerate these, it is useful to have a naming convention for the various doubly heavy hadrons. We will generically call the ground state a doubly heavy baryon with two Q quarks, Ξ_{QQ}, and the ground state of the tetraquark, T_{QQ}, which are analogous to the heavy (anti-) meson, $\overline{H}_Q$ (*i.e.*, the $\overline{D}$ and $\overline{B}$ mesons) and heavy Lambdas, Λ. We will use the following convention to indicate various types of hadron excitations:

* indicates an excitation of type (a);

$'$ indicates an excitation of type (b);

$^\sharp$ indicates an excitation of type (c).

In addition, we will indicate the DHDA equivalence between associating baryons and mesons.

Let us consider the phenomenological consequences of these types of excitations. In HQET, the $SU(2)$ heavy spin symmetry causes states which are only different by a spin flip to have the same mass. Excitations of type (a) are the type which will break this symmetry creating a mass difference between these states. For example, this will cause a mass difference between the spin-1 D^* meson and the spin-0 D meson. As this is the leading term to create the mass splitting, HQET dictates that this splitting is $O(\Lambda_H^2/M_Q)$ with corrections of $O(\Lambda_H^3/M_Q^2)$. Additionally, there are corrections to this hyperfine splitting due to pNRQCD. These corrections are related to the soft gluons that have been integrated out to construct the diquark potential. The leading corrections contribute at two loops, as shown in ref. [4], and are thus relative $O(\alpha_s^2)$. This implies in a correction to the mass splitting of $O(\Lambda_H^2\alpha_s^2/M_Q)$, which is formally larger than the $O(\Lambda^3/M_Q^2)$ corrections of HQET in the infinite mass limit. Because $\frac{\Lambda_H}{M_Q}$ is the smallest scale, these excitations should be the first excitations above the ground state.

Excitations of type (b) are all other excitations associated with the light degrees of freedom. These include orbital excitations between heavy and light components, as well as excitations within the light quark degrees of freedom. Due to the light quark mass, these excitations are in the non-perturbative regime of QCD, and can only be characterized by some general hadronic scale, Λ_H. Perturbative corrections to this are, in turn, meaningless. Traditional NRQCD has not been applied to systems with valance light quark degrees of freedom, and thus has ignored these excitations. HQET, on the other hand, combines these into the definitions of heavy fields from the outset, and thereby neglects them for the rest of the problem. We see here that the excitations should be qualitatively the second smallest scale.

Excitations of type (c) are internal diquark excitations. These excitations correspond to the excited levels of the color coulombic potential that binds the diquark. The binding potential is $V(r) = -\frac{2}{3}\frac{\alpha_s}{r}$, where the factor of $\frac{2}{3}$ comes from color considerations. This leads to energy levels and energy differences of:

$$E_n = -\frac{1}{9}\frac{\alpha_s^2 M_Q}{n^2}; \quad \Delta E = \frac{1}{12}M_Q\alpha_s^2 = \frac{1}{12}M_Q v^2. \tag{3}$$

The last step is justified since at the heavy quark scale, $\alpha_s(M_Q v) \sim v$. This verifies that type (c) excitations are $O(M_Q v^2)$. This type of excitation should be present in both the doubly heavy baryon and tetraquark sectors as the light quark interactions are suppressed since they are $O(\Lambda_H)$. This leads to mass relations such as:

$$\Xi_{cc}^\sharp - \Xi_{cc} = T_{cc}^{\Lambda\sharp} - T_{cc}^\Lambda = T_{cc}^\sharp - T_{cc} = \frac{1}{12}M_Q v^2 + O(M_Q v^4). \tag{4}$$

Since diquark excitations are $O(M_Q v^2)$, these are the largest excitations discussed here. The corrections to these relations can be found by considering the corrections to the color coulombic potential. In the context of NRQCD, it has been shown by [16] that these corrections are $O(M_Q v^4)$ at the heavy quark scale.

In addition to these excitations, DHDA symmetry will relate heavy mesons, $\bar{Q}q$ states, to doubly heavy baryons, QQq states, and relate heavy baryons, Qqq states, to doubly heavy tetraquarks, $\bar{Q}\bar{Q}qq$ states, which otherwise have the same quantum numbers. Therefore the following relations can be made:

$$\begin{aligned} D &\Leftrightarrow \Xi_{cc}; \\ D^* &\Leftrightarrow \Xi_{cc}^*; \\ \Lambda &\Leftrightarrow T_{cc}^\Lambda; \\ \Sigma, \Sigma^* &\Leftrightarrow T_{cc}, T_{cc}^*, T_{cc}^{**} \end{aligned} \tag{5}$$

where D and D^* are standard spin-0 and spin-1 D-mesons, Ξ_{cc} and Ξ_{cc}^* are spin-$\frac{1}{2}$ and spin-$\frac{3}{2}$ doubly heavy baryons, Λ is isospin-0 spin-$\frac{1}{2}$ heavy baryon, Σ and Σ^* are isospin-1 spin-$\frac{1}{2}$ and spin-$\frac{3}{2}$ heavy baryons, T_{cc}^Λ is a isospin-0 spin-0 doubly heavy tetraquark, T_{cc}, T_{cc}^{*}, T_{cc}^{**} are isospin-1 spin-0, spin-1, and spin-2 doubly heavy tetraquarks.

The DHDA symmetry can then be used to relate the mass splittings [1]. Equation (1) identifies the corrections to the mass splitting, but not to the DHDA symmetry itself. DHDA symmetry relies on the interactions between the heavy diquark and the light quark(s). These types of interactions, which are intrinsically non-perturbative, are not well understood in either NRQCD or HQET. Therefore, to understand the corrections to the symmetry, a new power counting scheme that combines the scales of NRQCD and HQET and is consistent with the other scales in the problem is necessary to account for these interactions systematically . At this time, such a system has not been formulated. Yet we can get a reasonable estimation of the corrections by considering the effects of the diquark structure compared with a point-like diquark on the DHDA symmetry. This consideration is exactly the form factor of the diquark relative to the scale of the light quark wave function. The form factor can be calculated by taking the Fourier transform of the square of the diquark wave function. In the limit of infinite heavy quarks, the diquark is in a coulombic wave function so the calculation is straightforward. Assuming that the momentum transferred is $O(\Lambda_H)$, the form factor can be expanded to give the leading correction to DHDA symmetry as follows:

$$F(q) \propto \frac{1}{(1+\frac{a_0^2}{4}q^2)^2} \sim 1-\frac{1}{2}a_0^2 q^2 \sim 1-\frac{1}{2}\frac{\Lambda_H^2}{M_Q^2(\frac{2}{3}\alpha_s)^2}, \quad (6)$$

where a_0 is the corresponding "Bohr radius" of the coulombic bound state of the diquark. Thus the corrections due to DHDA are $O(\Lambda_H^2/(M_Q^2\alpha_s^2))$. However, these corrections are formally smaller than the type (a) mass splitting correction of $O(\alpha_s^2)$. We can translate Eq. (1) into the previous notation, and extend the relations to include the tetraquark splittings to have:

$$\begin{aligned}
\Xi_{cc}^* - \Xi_{cc} &= \frac{3}{4}(D^* - D) + O(\Lambda_H^2\alpha_s^2/M_Q) \\
T_{cc}^{**} - T_{cc}^{*} &= \frac{4}{3}(\Sigma^* - \Sigma) + O(\Lambda_H^2\alpha_s^2/M_Q) \\
T_{cc}^{*} - T_{cc} &= \frac{2}{3}(\Sigma^* - \Sigma) + O(\Lambda_H^2\alpha_s^2/M_Q) \\
T_{cc} - T_{cc}^{\Lambda} &= 2(\Sigma - \Lambda) + O(\Lambda_H^2\alpha_s^2/M_Q)
\end{aligned} \quad (7)$$

where all quantities represent the mass of the corresponding particles.

To summarize, the doubly heavy baryons and tetraquarks will have three types of excitations which are distinct in the heavy quark limit. From these we can construct the hadronic spectrum for these particles based upon these excitations and their relative size to one another. Additionally, these spectra are related to the hadronic spectra of heavy mesons and heavy baryons via the DHDA symmetry. These spectra are presented in Figs. 2 and 3.

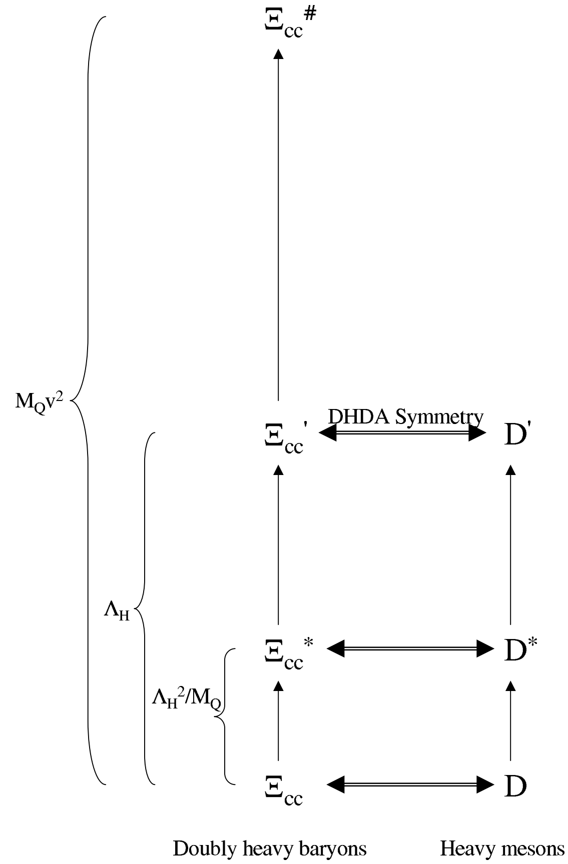

FIG. 2: Hadronic spectrum for doubly heavy baryons related to heavy mesons.

DHDA SYMMETRY AND THE PHYSICAL WORLD

In the previous section, we have worked solely in the infinite quark mass limit to determine what the spectrum would look like in this limit. We have seen the usefulness of DHDA symmetry in relating the the spectra of doubly heavy baryons to heavy mesons and doubly heavy tetraquarks to heavy baryons in this limit. We would like to use this tool to interpret the corresponding spectra with a finite massive heavy quark. As the heavy quark mass is decreased from infinity, we expect that the correction terms outlined above increase, until at a certain low enough quark mass, they become as dominant as the leading order resulting in a break down of the expansion. The discovery of doubly charmed baryons by the SELEX collaboration provides the first experimental data to verify the heavy hadronic spectrum described. An understanding of the SELEX data can provide an insight into whether DHDA symmetry persists in the real

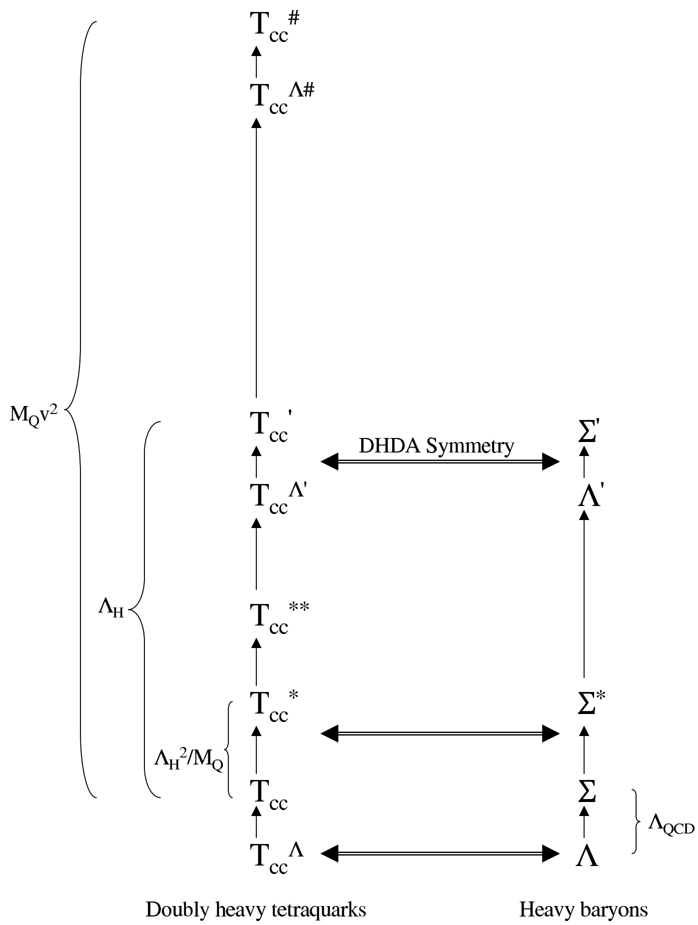

FIG. 3: Hadronic spectrum for doubly heavy tetraquarks related to heavy baryons.

world.

We can surmise that the SELEX data, along with real world parameters, could reveal one of three possible insights into the validity of DHDA symmetry for doubly charmed states. First, upon examining the data, we could find that the data supports a claim that the charm mass is heavy enough to be considered in the ideal large mass limit discussed in the previous section. If this were the case the spectrum can be easily interpreted in terms of an approximate DHDA symmetry. Secondly, the opposite could be true, namely that the SELEX data would be inconsistent with an approximate DHDA symmetry. This would indicate that the charm quark mass is simply too light for the symmetry to be manifest. The last possibility is perhaps the most interesting, that data could suggest that charm quark mass is not heavy enough for the preceding argument to hold in full, but that data would still be consistent with some aspects of an approximate DHDA symmetry. This last option is not unreasonable as the DHDA symmetry relies on the heavy diquark to be view as point-like with respect to the light degrees of freedom. The infinite mass limit ensures the validity of this assumption, but a small-sized diquark might be achieved even with a relatively modest heavy quark mass. For this possibility to be realized dynamics beyond the simple coulombic interaction must play a central role. To determine which of these possibilities is most consistent with the SELEX data, we will examine the size of each of the previously mentioned excitations, as well as their corrections, and compare them with experimentally determined parameters from the SELEX data.

Before doing this, we should note a general word of caution. The fact that the excited doubly charmed states were seen only via their weak decays presents a challenge to *any* simple interpretation of the data. The problem is that the electro-magnetic lifetime of the excited states as estimated by any simple model should be short enough to wash out any detection of excited states via their weak decay [17]. Any simple interpretation of the SELEX results cannot simultaneously understand the type of excitation that is observed as well as the lack of an electromagnetic decay channel. Therefore our focus here will be placing limitations on the type of excitation.

The excited state seen by SELEX shown in Fig. 1 could be interpreted as either a type (a) spin excitation or a type (c) diquark excitation. Type (b) light quark excitations are ruled out as they occur on the scale of hadronic physics which is much larger than the reported excitation. Either interpretation, as we will discuss, explains aspects of the data, but neither provides a complete explanation.

Scenario I: Spin excitation

Let us consider the case where the excited states are type (a) spin excitations. From our discussion of the infinite mass case, we would expect that even for a finite quark mass, these excitations would be the lowest lying occurring at $O(\Lambda_H^2/M_Q)$. According to the SELEX data, the excitation energy is 78MeV. With this identification, the DHDA mass splitting relations, Eq. (1) and Eq. (7) are satisfied with only a 30% deficiency as has been pointed out elsewhere [4, 5]. This size of error is also consistent with the equations' corrections of $O(\Lambda_H^2\alpha_s^2/M_Q)$. This appears to correspond to a success of DHDA symmetry.

At this point, our predecessors, [4, 5], have only verified that Eq. (1) is satisfied. This could be satisfied because DHDA symmetry is the underlying phenomenon or because of a numerical conspiracy. In order determine between these two scenarios, one needs to consider the other aspects of the spectrum and DHDA symmetry. That is, are type (c) excitation larger than type (a) excitation as expected when a finite quark mass is considered, and is the spatial extent of the diquark small enough to consider it point-like?

We will first tackle the former condition. For a diquark bound solely by color coulombic interactions, the excited state must by coulombic, and the excitation energy calculated from Eq. (3) is justified. From Eq. (3), we can calculate the expected excitation energy of the diquark for a charm quark mass of 1.15 GeV and a velocity of .53. This gives an excitation energy of 26.9 MeV! This is a clear sign that the scale separation arising from the color coulombic interactions, expected for an infinite quark mass, is not present for the charm quark.

The constrains on DHDA symmetry need to be examined. The key issue in determining whether DHDA symmetry could hold is the size of the diquark with regards to the light valance quark(s). This can be addressed by either looking at the size of the diquark to determine if it is nearly point-like, or to determine the size of corrections of DHDA symmetry as shown in Eq. (6). The size of the diquark can be characterized by the RMS radius of the state. For coulombic wave functions, the size of the diquark in the ground state is 1.64 fm. Clearly this is not point-like on the scale of hadronic physics. The large size of the ground state of the diquark also indicates that the excited state would be even larger. Such a spatially large excited state suggests that the excited state should extend beyond the color coulombic potential. This invalidates the previous calculation, while emphasizing the absurdity of assuming that the diquark is bound deeply by the color coulombic interaction. Moreover, this further indicates that the diquark must be under the influence of interactions in addition the the color coulombic potential. Additionally, the corrections to DHDA symmetry should be small compared with 1 if the approximation is used. For the values for the charm quark, the correction can be calculated, from Eq. (6) to be $3.02\frac{\Lambda_H^2}{\mathrm{GeV}^2}$, which for a typical hadronic scale of $\Lambda_H \sim 1$ GeV is not much smaller than 1. Thus both indicators show that the real world charm quark is not heavy enough to justify the point-like nature of the doubly heavy diquark which is necessary for the DHDA symmetry.

It should be noted that the bottom quark has a mass marginally large enough to approach the infinite mass limit scaling. The type (c) excitation is 32.8 MeV, with the type (a) excitation being 34.3 MeV calculated from the B-meson mass splitting. Additionally, the characteristic size is 0.79 fm, and the correction to the DHDA symmetry is $.69\frac{\Lambda_H^2}{\mathrm{GeV}^2}$. All of these numbers show that for the bottom quark the scale hierarchy is as expected and corrections are relatively small, even if the scale separation is not complete. However, presently doubly bottom baryons have not been observed experimentally.

We have shown that a naive approach to DHDA symmetry results in the conclusion that the charm quark is by no means heavy enough to believe that this symmetry is manifest in the real world, at least if it is to arise due to color coulombic interactions. In other words, the relatively small charm quark mass causes the corrections to the infinite massive limit to become large enough to question the expansion for the excited states. However, this does not completely rule out the possibility that DHDA could hold approximately and that these excitations are type (a). The color coulombic interactions are not the only interactions that the charm quarks could experience as part of a diquark or a doubly heavy baryon. Since the charm quarks are not heavy enough to fall into the color coulombic region, it is reasonable to surmise that these other non-perturbative interactions could conspire in such a manner that would facilitate an approximate DHDA symmetry. However, these additional non-perturbative interactions are not systematically included in NRQCD. Therefore, in order to describe this system, a new expansion that combines the perturbative and non-perturbative scales of NRQCD and HQET in a systematic manner is needed. At present, such an expansion has not yet been formulated. Nevertheless, by examining the properties of the interactions needed to maintain DHDA symmetry, a general picture of the new theory could be made.

Before proceeding with a discussion of the conditions that DHDA symmetry imposes on additional non-perturbative interactions, an additional comment on the color coulombic potential is needed. First, when we worked in the large mass limit, we were required to be in the regime of $M_Q v^2 \gg \Lambda_H$. However, with a finite massive quark this condition could be weakened to include $M_Q v^2 \sim \Lambda_H$. Under this condition, the type(b) and type (c) excitation may mix since they are at the same energy scale. Nevertheless, the key issue here is whether type (a) and type (c) excitations separate. The possible inclusion of type (b) excitations with type (c) does not effect whether they are separated from type (a), and hence do not effect the results discussed here. Secondly, the color coulombic potential is only the leading order term in NRQCD; sub-leading terms might need to be included when a finite massive quark is considered. However, since we have seen a need for a new expansion that includes the mixture of perturbative and non-perturbative effects, it is not clear whether the sub-leading terms suggested by NRQCD are the only sub-leading terms in the combined expansion. In both of these cases though, additional interactions beyond the simple color coulombic potential are included. It is not unreasonable that these, just like the ones hypothetically postulated above, would conspire so that the DHDA symmetry would be manifest in an approximate manner in the real world. Again, a description of the conditions to obtain an approximate DHDA symmetry will provide insight into these additional interactions whether they are NRQCD based or well beyond the scope of NRQCD and HQET.

There are two key places where the analysis based on the color coulombic potential fails to give rise to the DHDA symmetry with real world parameters. The assessment of these failures will provide conditions on the additional interactions to reestablish DHDA symmetry. The first is the characteristic size of the diquark. We have already shown that for the coulombic potential, the size of the diquark is large enough not to be considered even remotely point-like from the point of view of hadronic dynamics. Secondly, the hierarchy of scales used to derive the result breaks down badly. Additional dynamics beyond color coulombic would need to create a diquark with a size much smaller than the characteristic hadronic size and to re-establish the spin excitations as the lowest lying excitations as we originally assumed.

An examination of the restrictions placed on the characteristic size of the diquark reveals the following. The characteristic size of the diquark, which we will denote as L, must be smaller than the size in a coulombic potential, denoted L_c, and it must be small enough to allow the DHDA corrections to be small. The correction term of Eq. (6) can be rewritten in terms of this characteristic size as $\frac{1}{6}L^2\Lambda_H^2$. Thus for the correction to be small $L \ll \sqrt{6}/\Lambda_H \equiv L_{DHDA}$. L_c must be larger than L_{DHDA} since L_c already violates DHDA symmetry and thus cannot be smaller than L_{DHDA}. Therefore, in order for the diquark to be considered point-like, both

$$L \ll L_c \quad \text{and} \quad L \ll L_{DHDA} \tag{8}$$

must be simultaneously satisfied. In order to insure this, in terms of size, L_{DHDA} could be much smaller than L_c, or L_{DHDA} could be of comparable size to L_c. Consider the former possibility. $L_{DHDA} \ll L_c$ is equivalent to $\frac{\sqrt{6}}{\Lambda_H} \ll \frac{3}{2M_Q\alpha_s}$. This implies that $M_Q\alpha_s \ll .6\Lambda_H$. This relationship is never satisfied since $\alpha_s \sim 1/\ln(M_Q)$ and $M_Q \gg \Lambda_H$. Thus for DHDA symmetry to occur the latter condition must hold. It gives: $L_{DHDA} \sim L_c$ implying $M_Q\alpha_s \sim .6\Lambda_H$. As α_s at the charmed quark mass scale *is* around .6, this relation can only be satisfied if $M_Q \sim \Lambda_H$. It should be noted however, that an interaction that provides a characteristic size of the diquark which is consistent with Eq. (8) is possible. For the purposes of our discussion here, we needed to show that at least one kinematic region was possible, and the region where $M_Q \sim \Lambda_H$ satisfies these conditions even though it should not be unique.

A couple of comments should be made about this condition. The first is that naively appears not to occur even for the charm quark case. If one takes Λ_H to be of the scale of Λ_{QCD} it seems to be much smaller than M_c. However, we should note that the $\sim$ indicates "of the same scale as" under the assumption that the coefficients which arise in the expansion are "natural" *i.e.* of order unity. If the dynamics are such that some of the coefficients multiplying Λ_H are anomalously large, the condition $M_Q \sim \Lambda_H$ could hold effectively. The second key point is simply that if this does occur the system is clearly beyond the perturbative regime. It should also be noted that that this should *not* be seen as a generic condition invalidating NRQCD. Rather it implies that *for this particular system* the expansion has broken down. There is non-trivial evidence that this is in fact the case; namely if one assumes that the expansion is working one gets inconsistent results. The central question addressed here is not whether the expansion has broken down, rather it is whether one can still have a small diquark even if the expansion has broken down. If it indeed is the case that the condition $M_Q \sim \Lambda_H$ is effectively met, then there is a possible characteristic size of the charmed diquark, for which DHDA symmetry could be valid. This region is simply a size that is much smaller than the length associated with the coulombic potential and smaller than the typical hadronic size.

Thus far we have identified a possible kinetimatic region for which approximate DHDA symmetry may be possible. However, to test whether this can occur in practice, we need to see whether plausible dynamics can drive the system into such a regime. We do this by considering a "reasonable" dynamical model for the interaction between the heavy quarks. This model is not intended to be an accurate description of hadronic physics. The goal is simply to see whether a simple model with natural scales can put the system in the regime where DHDA symmetry emerges at least approximately. The existence of a model which does this shows that an approximate DHDA symmetry could be present in charm physics despite the fact that NRQCD in the coulombic regime plus HQET alone do not give rise to an approximate DHDA symmetry with the real world charm quark mass.

To illustrate the kind of model which brings us into this regime, we consider a linear confining potential with a string tension of $1\frac{\text{GeV}}{\text{fm}}$. Such a potential, with the same string tension, can be used to get a reasonable description of the J/Ψ [18]. One might not believe that such a model is applicable at all distances, to which we will attempt to apply. Indeed, one may reasonably question whether any two-body potential description is sensible. Nevertheless the scales of the model are at least instructive. Any confining potential that can be introduced will cause the characteristic size of the diquark to be reduced, thus the conditions on the diquark size may be satisfied. Specifically for the linear confining potential above binding charmed quarks, the characteristic length is 0.5 fm. This is substantially smaller than the coulombic wave function and might be small enough so that approximate DHDA might emerge. Moreover, the small size of the bound state helps to justify the two-body potential description *a posteriori*; the effects of the light quark between the heavy ones should be suppressed due to the small size. Unfortunately, this calculation is not part of a systematic calculation, and it is not immediately clear how to reliably estimate the size of the correction to the leading order DHDA estimate for the splitting.

Calculations of the energy spectrum of coulombic plus linear confining potentials in this channel reveals that the radial excitation energy is 630 MeV, far above the 100 MeV energy associated with expected spin excitations. Thus this linear confining potential satisfies both of the conditions needed to believe that an approximate DHDA symmetry could be realized for charmed quarks.

We have thus found a region where an approximate DHDA symmetry could be realized approximately and the lowest lying excitations are type (a) spin excitations. The color coulombic interactions cannot be the only relevant interactions that the heavy quarks experience (as is assumed in the heavy quark mass limit). Of course the question of whether the dynamics as such is realized in nature, remains an open question.

Even though we have provided a consistent argument for the observed excited states to be spin excitation, there remains a phenomenological issue with the parity of the excited state. Type (a) excitations do not change the parity of the excited state relative to the ground state. Ground state baryons have positive parity, thus the spin excited state should also have positive parity. Experimentally, the parity of the excited states has not been determined. The SELEX collaboration have argued that the orbital angular momentum of the ground state is consistent with $L = 0$ (positive parity), while the excited state is consistent with $L > 0$ (either positive or negative parity). Furthermore, SELEX observed an orbital excited state $\Xi_{cc}(3780)$ which has negative parity and decays via pion emission to $\Xi_{cc}(3520)$ suggesting that this state could have negative parity. If this parity assignment holds, the interpretation that the excited states were spin excitations, made here and in refs. [4, 5], would be ruled out.

Scenario II: Diquark Excitation

Now let us consider the case where the excitation is interpreted as a type (c) diquark excitation. Type (c) excitations could result in a parity flip from the ground state. This would resolve the parity problem found with the spin excitation interpretation. As we will discuss below, if this scenario is correct we are almost certainly outside the regime of validity of DHDA as well as outside the regime of validity of NRQCD. Moreover, it is likely to be very difficult to make such a scenario work phenomenologically.

In order for diquark excitations to be smaller than the spin excitations, there must be a break down of the heavy quark mass limit; the system must reside in a non-perturbative regime. Therefore, as with the previous case, the diquark can be under the influence of non-perturbative interactions beyond the color coulombic interactions. We showed that if these additional confining interactions maintained an approximate DHDA symmetry, the diquark excitations were much larger than the observed 78 MeV excitation. Again we can illustrate this with a linearly rising potential between the heavy quarks. In order for the diquark excitations to be comparable to the observed splitting, the linear confining interactions must have a string constant of $\sim 50\frac{\text{MeV}}{\text{fm}}$, which is very small compared to the natural scales in the problem. The small size of the linear confining interactions results in the diquark having a larger size, and makes the assumptions that it is point-like even less believable. For the string constant of 50 $\frac{\text{MeV}}{\text{fm}}$ considered here, the ground state of the diquark has an RMS radius of 1.2 fm, and the first excited state has an RMS radius of 5.8 fm. These numbers are extremely large compared to typical hadronic sizes.

The preceding model calculation suggests that if the excitation were the excitation of the diquark, the DHDA symmetry cannot be valid even approximately. It also raises a fundamental issue of self-consistency. A large spatially extent diquark allows the light quark to come between the two heavy quarks allowing for three-body interactions to play a significant dynamical role. To the extent that this occurs, it is meaningless as a phenomenological matter to separate the diquark excitation from excitations of the entire system. Thus, excitations of type (b) and (c) would strongly mix and the entire structure of scale separation would break down.

We note that this analysis was based on a very simple and not terribly plausible model. However, it does incorporate the natural scales of the problem and shows that the excited state wave functions are *much* too large to be taken seriously. It is also clear that it would be very hard to construct any potential model which restricts the diquark size to be much less than a fermi while having an excitation energy of 78 MeV. To illustrate this point, we can consider a harmonic confining potential instead of the linear potential. We would expect that this potential would confine the excited state and reduce its size more than the linear potential. Calculating the size of the diquark under these conditions for an excitation energy of 78 MeV results in a ground state RMS radius of 1.3 fm and an excited state RMS radius of 2.5 fm. Even though the diquark size is smaller, it is still very large in terms of hadronic physics. Furthermore, if we were able to drive the size of the excited state to a reasonable hadronic size, say, 1 fm, the ground state would be even smaller. Such a small ground state size is then consistent with spin excitations discussed previously. It thus seems difficult for this scenario to be correct.

Together these two scenarios make it very difficult to understand the data in a simple way. If the parity of the is the parity of the states is correctly interpreted by SELEX, there does not appear to be any simple phenomoenologically reasonable interactions yielding either small diquark excitations or DHDA symmetry. However, if we set the parity designation aside, scenario I, the assignment of type (a) spin excitations, seems to be the more plausible interpretation.

CONCLUSION

The SELEX data on doubly heavy baryons is very difficult to interpret. As noted in the introduction, the fact the excited states were detectable through their weak decays when there were open channels for electromagnetic decays is very problematic; normally one would expect these to dilute the strength to the point that the states would be very difficult to see. Despite this problem, we have attempted to understand the SELEX states in the context of DHDA symmetry. We have shown that the data is not consistent with the heavy quark mass limit, but this does not rule out an approximate DHDA symmetry. This could emerge if diquark interactions beyond color coulombic interactions are considered. As such, a new systematic expansion, which is a hybrid of HQET and NRQCD but outside the domain of the color coulombic, would be the most appropriate to describe the physics of doubly heavy baryons. Such an expansion could help in the understanding of the SELEX observations. At the same time it is critical to add to our understanding of the experimental situation. In particular, it is essential that the observed states be confirmed in other experiments; that the parity of the states are pinned down; and the accurate measurements of electromagnetic transitions are made. These measurements are critical in understanding the doubly charm baryon spectrum as well as the validity of DHDA symmetry in charm physics.

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